

# Quantitative Methods

## LEARNING OUTCOMES

### **The Time Value of Money**

*The candidate should be able to:*

- interpret interest rates as required rates of return, discount rates, or opportunity costs
- explain an interest rate as the sum of a real risk-free rate and premiums that compensate investors for bearing distinct types of risk
- calculate and interpret the future value (FV) and present value (PV) of a single sum of money, an ordinary annuity, an annuity due, a perpetuity (PV only), and a series of unequal cash flows
- demonstrate the use of a time line in modeling and solving time value of money problems
- calculate the solution for time value of money problems with different frequencies of compounding
- calculate and interpret the effective annual rate, given the stated annual interest rate and the frequency of compounding

### **Organizing, Visualizing, and Describing Data**

*The candidate should be able to:*

- identify and compare data types
- describe how data are organized for quantitative analysis
- interpret frequency and related distributions
- interpret a contingency table
- describe ways that data may be visualized and evaluate uses of specific visualizations

- describe how to select among visualization types
- calculate and interpret measures of central tendency
- evaluate alternative definitions of mean to address an investment problem
- calculate quantiles and interpret related visualizations
- calculate and interpret measures of dispersion
- calculate and interpret target downside deviation
- interpret skewness
- interpret kurtosis
- interpret correlation between two variables

### **Probability Concepts**

*The candidate should be able to:*

- define a random variable, an outcome, and an event
- identify the two defining properties of probability, including mutually exclusive and exhaustive events, and compare and contrast empirical, subjective, and a priori probabilities
- describe the probability of an event in terms of odds for and against the event
- calculate and interpret conditional probabilities
- demonstrate the application of the multiplication and addition rules for probability
- compare and contrast dependent and independent events
- calculate and interpret an unconditional probability using the total probability rule
- calculate and interpret the expected value, variance, and standard deviation of random variables
- explain the use of conditional expectation in investment applications
- interpret a probability tree and demonstrate its application to investment problems
- calculate and interpret the expected value, variance, standard deviation, covariances, and correlations of portfolio returns
- calculate and interpret the covariances of portfolio returns using the joint probability function
- calculate and interpret an updated probability using Bayes' formula
- identify the most appropriate method to solve a particular counting problem and analyze counting problems using factorial, combination, and permutation concepts

### **Common Probability Distributions**

*The candidate should be able to:*

- define a probability distribution and compare and contrast discrete and continuous random variables and their probability functions
- calculate and interpret probabilities for a random variable given its cumulative distribution function
- describe the properties of a discrete uniform random variable, and calculate and interpret probabilities given the discrete uniform distribution function
- describe the properties of the continuous uniform distribution, and calculate and interpret probabilities given a continuous uniform distribution
- describe the properties of a Bernoulli random variable and a binomial random variable, and calculate and interpret probabilities given the binomial distribution function
- explain the key properties of the normal distribution
- contrast a multivariate distribution and a univariate distribution, and explain the role of correlation in the multivariate normal distribution

- calculate the probability that a normally distributed random variable lies inside a given interval
- explain how to standardize a random variable
- calculate and interpret probabilities using the standard normal distribution
- define shortfall risk, calculate the safety-first ratio, and identify an optimal portfolio using Roy's safety-first criterion
- explain the relationship between normal and lognormal distributions and why the lognormal distribution is used to model asset prices
- calculate and interpret a continuously compounded rate of return, given a specific holding period return
- describe the properties of the Student's  $t$ -distribution, and calculate and interpret its degrees of freedom
- describe the properties of the chi-square distribution and the  $F$ -distribution, and calculate and interpret their degrees of freedom
- describe Monte Carlo simulation

### Sampling and Estimation

#### *The candidate should be able to:*

- compare and contrast probability samples with non-probability samples and discuss applications of each to an investment problem
- explain sampling error
- compare and contrast simple random, stratified random, cluster, convenience, and judgmental sampling
- explain the central limit theorem and its importance
- calculate and interpret the standard error of the sample mean
- identify and describe desirable properties of an estimator
- contrast a point estimate and a confidence interval estimate of a population parameter
- calculate and interpret a confidence interval for a population mean, given a normal distribution with 1) a known population variance, 2) an unknown population variance, or 3) an unknown population variance and a large sample size
- describe the use of resampling (bootstrap, jackknife) to estimate the sampling distribution of a statistic
- describe the issues regarding selection of the appropriate sample size, data snooping bias, sample selection bias, survivorship bias, look-ahead bias, and time-period bias

### Hypothesis Testing

#### *The candidate should be able to:*

- define a hypothesis, describe the steps of hypothesis testing, and describe and interpret the choice of the null and alternative hypotheses
- compare and contrast one-tailed and two-tailed tests of hypotheses
- explain a test statistic, Type I and Type II errors, a significance level, how significance levels are used in hypothesis testing, and the power of a test
- explain a decision rule and the relation between confidence intervals and hypothesis tests, and determine whether a statistically significant result is also economically meaningful
- explain and interpret the  $p$ -value as it relates to hypothesis testing
- describe how to interpret the significance of a test in the context of multiple tests

- identify the appropriate test statistic and interpret the results for a hypothesis test concerning the population mean of both large and small samples when the population is normally or approximately normally distributed and the variance is (1) known or (2) unknown
- identify the appropriate test statistic and interpret the results for a hypothesis test concerning the equality of the population means of two at least approximately normally distributed populations based on independent random samples with equal assumed variances
- identify the appropriate test statistic and interpret the results for a hypothesis test concerning the mean difference of two normally distributed populations
- identify the appropriate test statistic and interpret the results for a hypothesis test concerning (1) the variance of a normally distributed population and (2) the equality of the variances of two normally distributed populations based on two independent random samples
- compare and contrast parametric and nonparametric tests, and describe situations where each is the more appropriate type of test
- explain parametric and nonparametric tests of the hypothesis that the population correlation coefficient equals zero, and determine whether the hypothesis is rejected at a given level of significance
- explain tests of independence based on contingency table data

### **Introduction to Linear Regression**

#### ***The candidate should be able to:***

- describe a simple linear regression model and the roles of the dependent and independent variables in the model
- describe the least squares criterion, how it is used to estimate regression coefficients, and their interpretation
- explain the assumptions underlying the simple linear regression model, and describe how residuals and residual plots indicate if these assumptions may have been violated
- calculate and interpret the coefficient of determination and the  $F$ -statistic in a simple linear regression
- describe the use of analysis of variance (ANOVA) in regression analysis, interpret ANOVA results, and calculate and interpret the standard error of estimate in a simple linear regression
- formulate a null and an alternative hypothesis about a population value of a regression coefficient, and determine whether the null hypothesis is rejected at a given level of significance
- calculate and interpret the predicted value for the dependent variable, and a prediction interval for it, given an estimated linear regression model and a value for the independent variable
- describe different functional forms of simple linear regressions